Derivative Tables

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**Questions in past papers often come up combined with other topics.
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).**

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Question 1

Qualification: AP Calculus AB

Areas: Integration, Applications of Differentiation, Limits and Continuity

Subtopics: Properties of Integrals, Fundamental Theorem of Calculus (First), Concavity, Tangents To Curves, Mean Value Theorem, Continuities and Discontinuities, Derivative Tables

Paper: Part B-Non-Calc / Series: 2002 / Difficulty: Hard / Question Number: 6

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

6. Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

(a) Evaluate $\int_0^{1.5} (3f'(x) + 4)\,dx$. Show the work that leads to your answer.

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

(c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

(d) Let g be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \ge 0. \end{cases}$

The graph of g passes through each of the points $(x, f(x))$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.

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Question 2

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation, Integration

Subtopics: Local or Relative Minima and Maxima, Derivative Tables, Derivative Graphs, Fundamental Theorem of Calculus (Second), Points Of Inflection, Increasing/Decreasing

Paper: Part B-Non-Calc / Series: 2005 / Difficulty: Somewhat Challenging / Question Number: 4

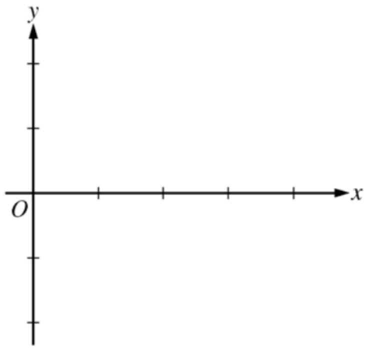
x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

4. Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of f.

(Note: Use the axes provided in the pink test booklet.)



(c) Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.

(d) For the function g defined in part (c), find all values of x, for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

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Question 3

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Integration

Subtopics: Interpreting Meaning in Applied Contexts, Derivative Tables, Kinematics (Displacement, Velocity, and Acceleration), Riemann Sums – Trapezoidal Rule, Intermediate Value Theorem, Mean Value Theorem

Paper: Part B-Non-Calc / Series: 2006-Form-B / Difficulty: Hard / Question Number: 6

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft / sec)	–20	–30	–20	–14	–10	0	10
$a(t)$ $\left(\text{ft} / \text{sec}^2\right)$	1	5	2	1	2	4	2

6. A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

(a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|\, dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|\, dt$ using a trapezoidal approximation with the three subintervals determined by the table.

(b) Using appropriate units, explain the meaning of $\int_{0}^{30} a(t)\, dt$ in terms of the car's motion. Find the exact value of $\int_{0}^{30} a(t)\, dt$.

(c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.

(d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

Question 4

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation, Integration

Subtopics: Intermediate Value Theorem, Mean Value Theorem, Fundamental Theorem of Calculus (Second), Tangents To Curves, Derivative Tables

Paper: Part A-Calc / Series: 2007 / Difficulty: Very Hard / Question Number: 3

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

3. The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

   (a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

   (b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

   (c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t)\,dt$. Find the value of $w'(3)$.

   (d) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

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Question 5

Qualification: AP Calculus AB

Areas: Integration

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Interpreting Meaning in Applied Contexts, Riemann Sums – Trapezoidal Rule, Derivative Tables, Rates of Change (Average)

Paper: Part B-Non-Calc / Series: 2009-Form-B / Difficulty: Somewhat Challenging / Question Number: 6

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	−10	−8	−4	7

6. The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

(a) Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{20}^{40} v(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)\,dt$.

(c) For $0 \le t \le 40$, must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

(d) Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.

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Question 6

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Mean Value Theorem, Local or Relative Minima and Maxima, Derivative Tables, Differentiation Technique – Chain Rule, Fundamental Theorem of Calculus (First)

Paper: Part B-Non-Calc / Series: 2014 / Difficulty: Somewhat Challenging / Question Number: 5

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

5. The twice-differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values of x are given in the table above.

(a) Find the x-coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.

(b) Explain why there must be a value c, for $-1 < c < 1$, such that $f''(c) = 0$.

(c) The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.

(d) Evaluate $\int_{-2}^{3} f'(g(x))g'(x)\,dx$.

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Question 7

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Derivative Tables, Tangents To Curves, Differentiation Technique - Quotient Rule, Fundamental Theorem of Calculus (First), Differentiation Technique – Chain Rule

Paper: Part B-Non-Calc / Series: 2016 / Difficulty: Medium / Question Number: 6

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	–6	3	2	8
2	2	–2	–3	0
3	8	7	6	2
6	4	5	3	–1

6. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \frac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x)\,dx$.

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Question 8

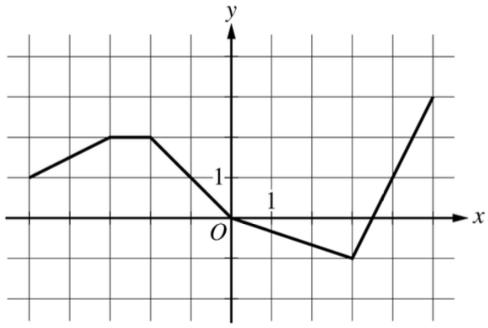
Qualification: AP Calculus AB

Areas: Differentiation, Applications of Differentiation

Subtopics: Derivative Tables, Tangents To Curves, Differentiation Technique – Chain Rule, Derivative Graphs, Differentiation Technique – Product Rule, Mean Value Theorem, Differentiation Technique – Trigonometry, Differentiation Technique – Exponentials

Paper: Part B-Non-Calc / Series: 2017 / Difficulty: Medium / Question Number: 6

x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

(c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

(d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

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Question 9

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Derivative Tables, Differentiation Technique – Chain Rule, Concavity, Differentiation Technique – Product Rule, Fundamental Theorem of Calculus (First), Increasing/Decreasing

Paper: Part B-Non-Calc / Series: 2023 / Difficulty: Somewhat Challenging / Question Number: 5

x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	-8	3	6
$g(x)$	1	2	-3	0
$g'(x)$	5	4	2	8

5. The functions f and g are twice differentiable. The table shown gives values of the functions and their first derivatives at selected values of x.

(a) Let h be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer.

(b) Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer.

(c) Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t)\, dt$. Find $m(2)$. Show the work that leads to your answer.

(d) Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer.